Life Is Not Fair

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(Un)Fairness Function

Take any standard variant of P systems and a standard derivation mode.

The application of a multiset of rules in addition can be guided by a function computed based on specific features of the underlying configuration of the multiset of rules applicable to this configuration.

The choice of the multiset of rules to be applied then depends on the function values computed for all the applicable multisets of rules.

(Un)Fairness Function

One may argue that it is fair to use rules in such a way that each rule should be applied if possible and as well equally often.

Hence, a fairness function for applicable multisets should compute the best value for those multisets of rules fulfilling these guidelines.

On the other hand, we may choose the multiset of rules to be applied in such a way that it is the unfairest one ;-)

An Unfair Example

If a rule is applied n times then it contributes to the function value of the fairness function for the multiset of rules with 2^{-n}.

Take a P system with one membrane working in the maximally parallel way, starting with the axiom b and using the two rules 1:b➔bb and 2:b➔a. If we apply only one of these rules m times to all objects b, then the function value is 2^{-m} and is minimal compared to the function values computed for a mixed multiset of rules using both rules at least once.

An Unfair Example

Starting with the axiom b we use the rule 1:b➔bb in the maximal way k times thus obtaining 2^k symbols b. Then in the last step, for all b we use the rule 2:b➔a thus obtaining 2^k symbols a.

We cannot mix the two rules in one of the derivation steps as only the clean use of exactly one of them yields the minimal value for the fairness function.

We observe that the effect is similar to that of controlling the application of rules by label control.

A Weird Example

Take a P system with one membrane working in the maximally parallel way, starting with the axiom b and using the three rules 1:b→bb, 2:b→b and 3:b→a. Moreover let M be an arbitrary set of positive natural numbers. The fairness function on multisets of rules over these three rules and a configuration containing m symbols b is defined as

- 0 if we only use rule 3 and m is in M,
- 0 if we use rule 1 once and rule 2 for the rest,
- 1 for any other multiset of rules.

A Weird Example

If we use rule 1:b→bb once and rule 2:b→b for the rest, this increases the number of symbols b in the skin membrane by one. Thus, in m-1 steps we get m symbols b. If m is in M, we now may use rule 3:b→a for all symbols b, thus obtaining m symbols a, and the system halts. In that way, the system generates exactly $\{a^m : m \text{ in } M\}$.

To make this example a little bit less weird, we may only allow computable sets M. Still, the whole computing power is in the fairness function alone.

Simulating Priorities in the Sequential Derivation Mode

In the sequential derivation mode, exactly one rule is applied in every derivation step of the P system Π. Given a configuration C and the set of applicable rules Appl(Π,C) not taking into account a given priority relation < on the rules, the fairness function yields 0 for each rule in Appl(Π,C) for which no rule in Appl(Π,C) with higher priority exists and 1 otherwise. Thus, only rules with highest priority can be applied.

Simulating Energy Control

Recently we have considered P systems where a specific amount of energy is assigned to each rule.

Only those multisets of rules are applied which use the minimal amount of energy.

In a similar way the amount of energy coming up with a multiset of rules can be seen as the value of the fairness function. The minimal amount of energy then exactly corresponds with the minimal fairness.