Notes on the power of some restricted variants of P systems with active membranes

Zsolt Gazdag, Gábor Kolonits

Faculty of Informatics,

Eötvös Loránd University

Motivation

- P systems with active membranes are best known for their ability to solve NP-complete problems in polynomial time
- It is also interesting which combination of the possible features are **not** enough to solve NP-complete problems, but enough to solve problems in P
- For example, elementary membrane division rules are necessary to solve NP-complete problems [2000, Zandron, Fernetti, Mauri]
- But not sufficient to solve problems in P if polarizations and dissolution rules are not allowed [2008, Murphy, Woods] (based on the notion of *dependency graph* introduced in [2006, Gutierrez-Naranho et al.])

Motivation

- Unfortunately, showing P lower and upper bounds on the power of certain variants of P systems with active membranes is not always easy
- The P conjecture: P systems with active membranes using no polarizations characterize the class P [2005, Paun]
  - P lower bound is already proved (semi uniform solution in [2011, Murphy, Woods], uniform solution in [2014, Gazdag et al.]
  - P upper bound is still unproved

Motivation

- There are positive results for the P upper bound in restricted cases, for example when
  - only symmetric elementary division is allowed [2007, Murphy, Woods] or when
  - polarization, evolution rules and communications rules are not allowed and the system has a restricted initial membrane structure [2009, Woods et al.]
- On the other hand, it seems to be very difficult to solve a P-complete problem with P systems with no polarizations, when evolution and communication rules are not allowed
- In this talk we discuss the lower and upper bounds of the computational power of certain restricted variants of P systems with active membranes

Preliminaries

- P systems with active membranes have the following types of rules
  - Evolution rules
  - In and out communication rules
  - Dissolution rules,
  - Membrane division rules
    - in this talk we do not consider those systems which employ non-elementary division rules, they can decide PSPACE complete problems even without polarization, evolution and communication [2009, Zandron et al.]
- We assume the usual maximal parallel derivation strategy

Preliminaries

- Recognizer P systems
  - Every computation halts and yields the same answer yes or no,
  - The input is placed into a designated *input membrane*
  - The output appears in the last step of the computation in a designated output membrane
- To solve decision problems we use *uniform* families of recognizer P systems
- In this talk we consider possible solutions of the NL-complete STCON problem:
  - Given a directed graph $G = (V, E)$ and $s, t \in V$
  - Decide if there is a path from s to t

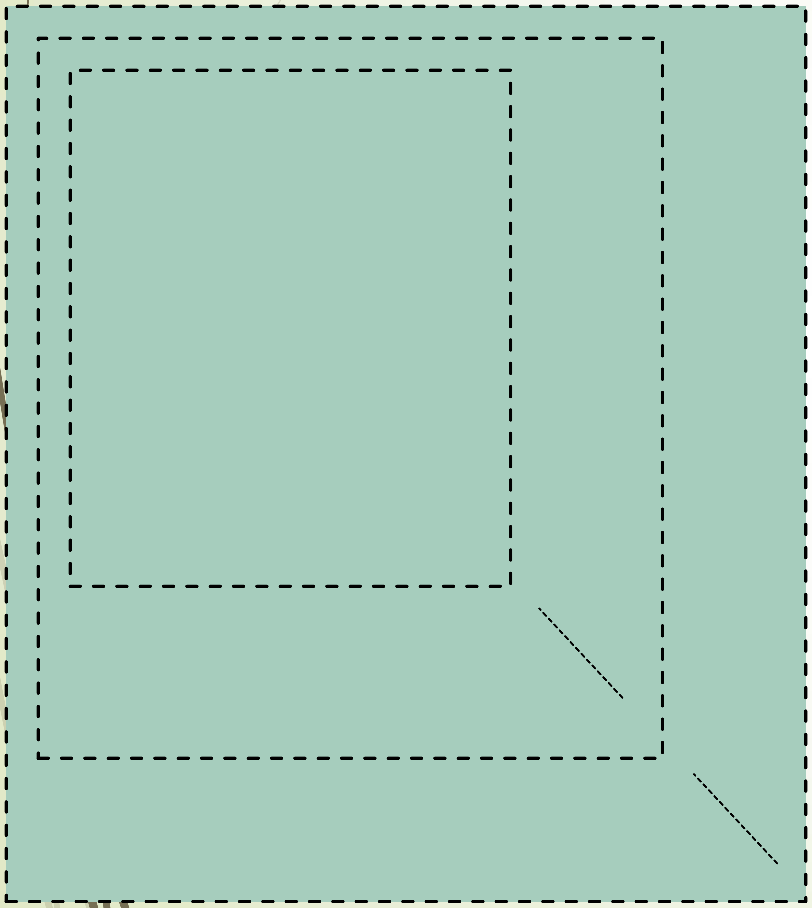
Method for solving STCON

- Given a directed graph $G = (V, E)$ and $s, t \in V$
  - Decide if there is a path from s to t
  - {We may assume, without the loss of generality, that our vertices are labeled with natural numbers from 1 to N, and $s = 1$ and $t = N$}
- We compute a set H of vertices step-by-step
  - Initially, H contains only 1
  - In every step, we add to H those vertices that are reachable from the elements currently in H
  - After at most $N - 1$ steps every vertices reachable from 1 are present in H

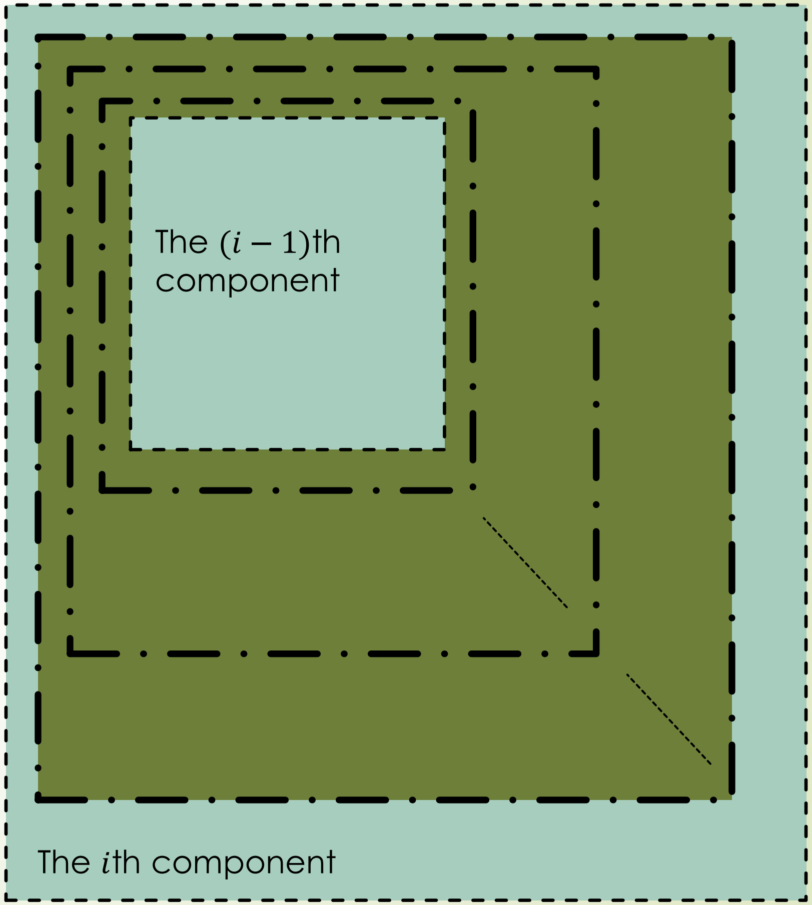
Possible simulation of this method

- We use an encapsulated structure
  - There are $N-1$ ***components***, one for each main step of the computation of H
  - In every component, there are $N*(N-1)$ ***layers***, one for every pair of vertices (every possible edges)
  - Every layer may introduce a new vertex
  - The innermost layer contains the next component

The structure



The ***components***



The ***layers*** and the next ***component***

Cases to handle during the step-by-step computation

- There can be several cases during the computation involving the vertices i and j
  - There is **no** edge from i to j $(case\ 1)$
  - There is an edge from i to j, but i is **not** present in H after $k-1$ steps $(case\ 2)$
  - There is an edge from i to j, i is present in H after $k-1$ steps, but j is **not** $(case\ 3)$
  - There is an edge from i to j, and both i and j are present in H after $k-1$ steps $(case\ 4)$

A solution without evolution, dissolution, division, and in-communication rules

- We describe a uniform family of P systems to implement the above method solving STCON
  - Encoding of the input
    - An object ij represents, that there is a directed edge from the vertex i to j
    - An object $\overline{ij}$ represents, that there isn't such an edge
    - We call them ***positive*** and ***negative edge-objects***
  - Every layer consists two membranes, labeled with i, j, a and i, j, b [the layer is associated with the pair of vertices (i, j)]
  - The input membrane is the innermost
    - It contains an object 1 and objects $\overline{2}, \overline{3}, \ldots, \overline{N}$ (representing, that initially only vertex 1 is reachable
    - We call them ***positive*** and ***negative vertex-objects***)
  - Initially every membrane has neutral charge

A solution without evolution, dissolution, division, and in-communication rules

- During the computation the following invariant properties will hold
  - $\forall\, i \in [1 \ldots N]$: exactly one of the vertex-objects i or $\bar{i}$ is present in the system
  - After going through the layers of the kth component it correctly represents that i can be reached from 1 in at most k steps
- Main rule: every object can go through every membrane that has negative polarization

Initializing the layers

- First, the edge-objects set the polarizations of the corresponding i, j, a membranes by out-communication rules
  - A positive [resp. negative] edge-object sets the polarization to positive [resp. negative])
  - The i, j, b membranes keep their neutral charges
- Then the vertex-objects can begin their „journey" to the $SKIN$

Handle the possible cases

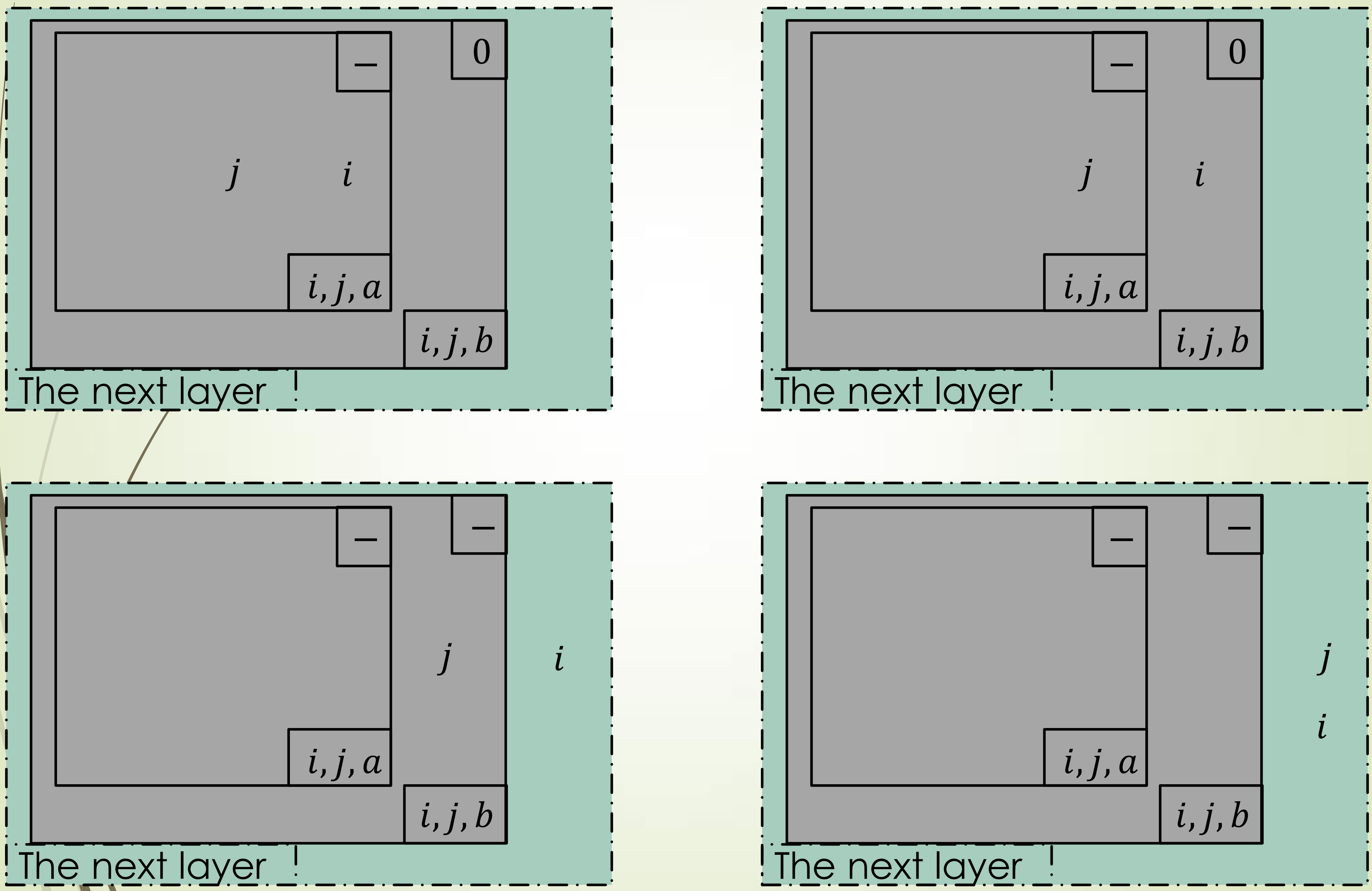
- If a positive (resp. negative) vertex-object i (resp. $\bar{i}$) reaches an i, j, a membrane with negative polarization, the object goes through it, then goes through the i, j, b membrane with neutral charge, and sets its polarization to negative (case 1)
- If a negatvie vertex-object $\bar{i}$ reaches an i, j, a membrane with positive charge, the object goes through it, and sets its polarization to negative , then goes through the i, j, b membrane with neutral charge, and sets its polarization to negative too (case 2)

Handle the possible cases

- If a positive vertex-object i reaches an i, j, a membrane with positive charge, the object goes through it and „grabs" that charge (it becomes an i^+ vertex-object, and the membranes polarization becomes negative)
- The i^+ object then „gives" its charge to the i, j, b membrane, and becomes an i vertex-object again
- Then, if a $\bar{j}$ negative vertex-object comes to the i, j, b membrane (with positive polarization) it goes out, „grabs" the charge, and becomes a j vertex-object, and the membranes polarization becomes negative (case 3)
- If a j positive vertex-object meets the membrane, it sets its polarization to negative too (case 4)

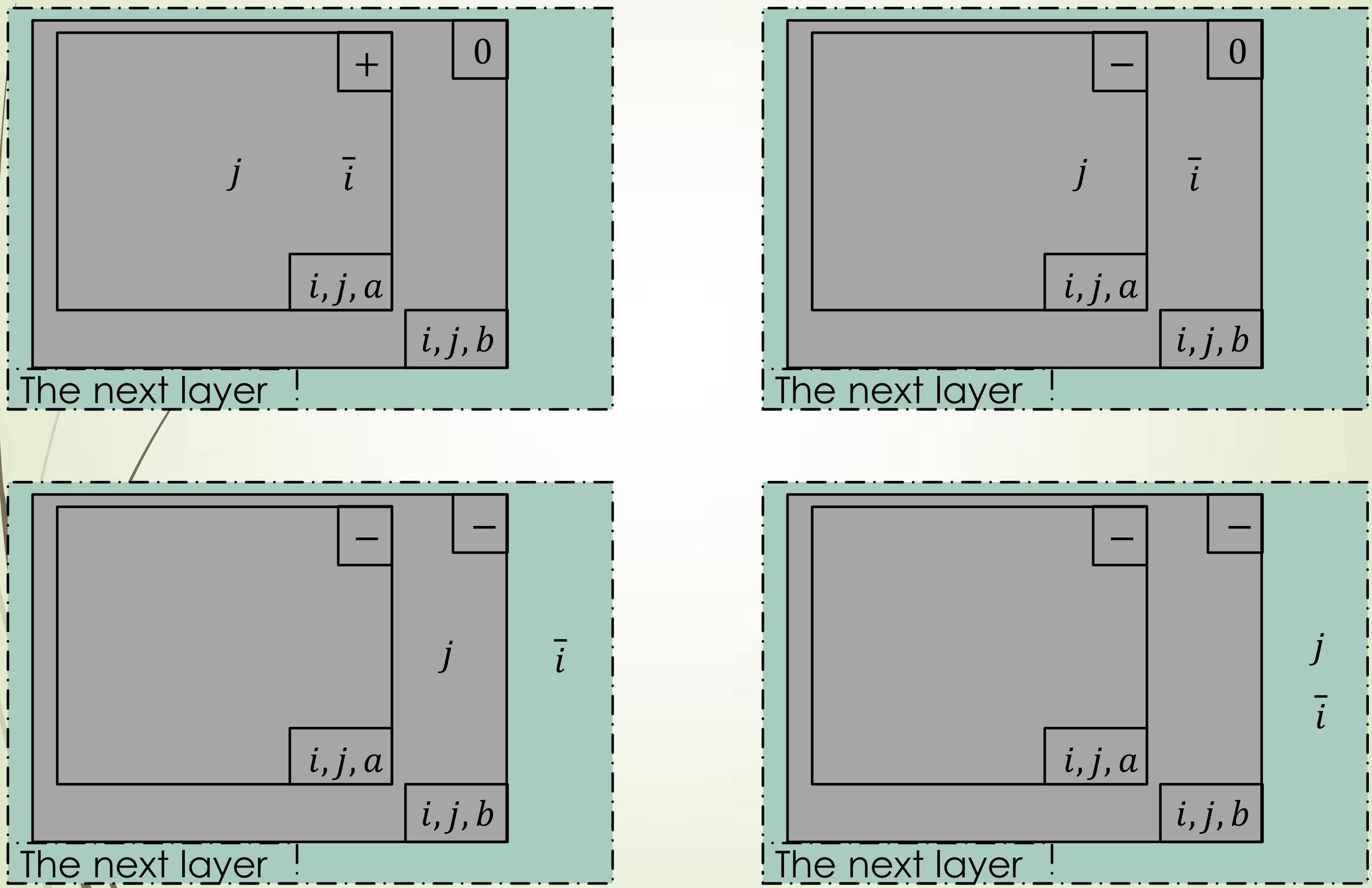
Handle the possible cases

Case 1



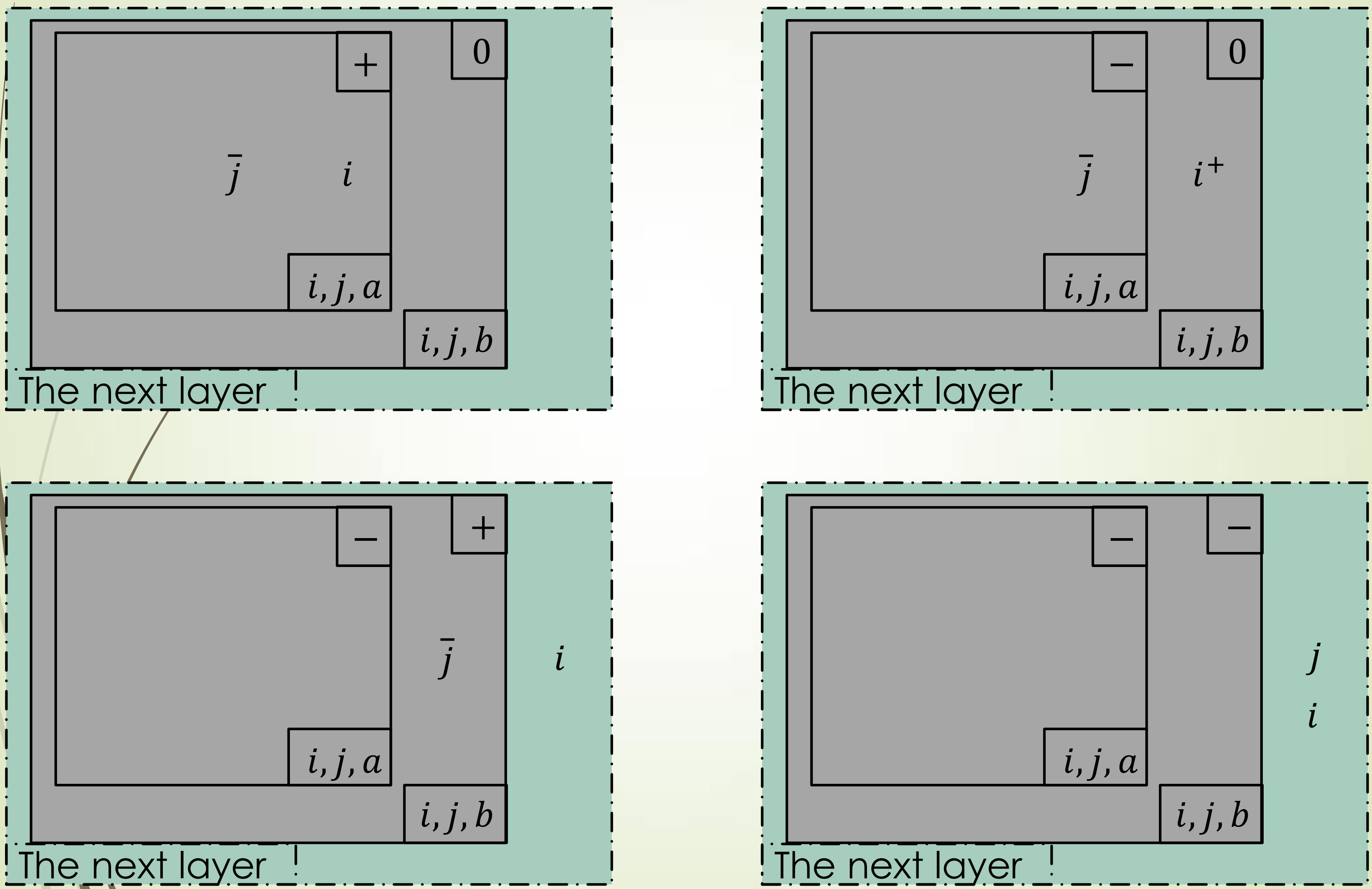
Handle the possible cases

Case 2



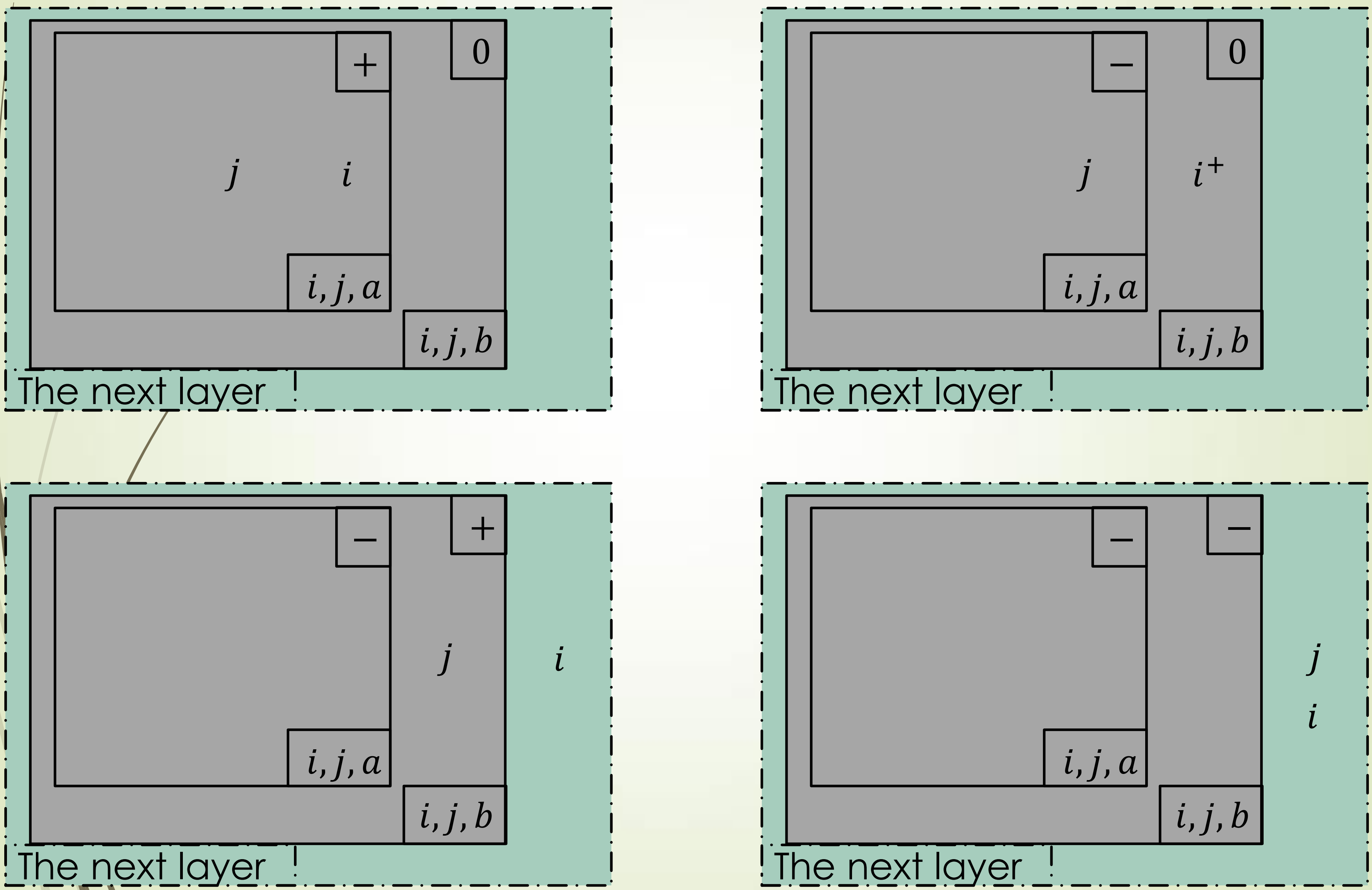
Handle the possible cases

Case 3



Handle the possible cases

Case 4



A solution without evolution, and communication rules

- We use a very similar system, with some minor changes
  - We use positive and negative edge- and vertex-objects
  - Minor change in the invariant property: we have exactly one negative vertex-object, or at least one positive vertex-object to a vertex
  - Trivially, the „main rule" won't hold ☺
  - Exchanging a negative vertex-object to a positive variant is made with a sequence of elementary divisions and dissolutions
  - During it, we create unnecessary copies of negative vertex-objects, that must be removed, so we must extend the layers with ***removers***

An attempt to increase the lower bound to P

- Horn formula: a propositional formula φ in the conjunctive normal form (CNF) such that every clause of φ contains at most one positive literals
  - E.g. $x \lor \neg y \lor \neg z$, x, $\neg y$ are Horn clauses
- HORNSAT: Given a Horn formula φ, decide if φ is satisfiable
- It is known that HORNSAT is P-complete
- The direct solution of HORNSAT seems to be difficult due to the very limited ability of communication
- We consider HORN3SAT: Given a Horn formula φ such that every clause of φ contains at most three literals; Decide if φ is satisfiable

An attempt to increase the lower bound to P

- HORNSAT $\leq_l$ HORN3SAT:
  - For a Horn formula φ, a HORN3SAT instance φ' can be constructed using logarithmic space s.t.
    φ is satisfiable iff φ' is satisfiable
  - Example: $C = x \vee \neg y \vee \neg z \vee \neg u \in \varphi \Rightarrow$
  - $C_1 = x \vee \neg y \vee \neg n$ and $C_2 = n \vee \neg z \vee \neg u \in \varphi'$ (n is a new propositional variable)
  - C_1 and C_2 are Horn clauses
- Thus HORN3SAT is P-complete
- Observation: $x \vee \neg y \vee \neg z \sim y \wedge z \to x, \;\; x \sim \uparrow \to x, \;\; \neg y \sim y \to \downarrow$

An attempt to increase the lower bound to P

- Recall: in case of STCON the presented P systems computed a set of those vertices that are reachable from s
  - This was done step by step: given the set of those vertices that can be reached from s in at most i steps, the P systems computed the set of those vertices that can be reached from s in at most $i+1$ steps.
  - Basically, the systems followed the edges of the form $u \to v$ represented by the membranes
- In case of HORN3SAT the P systems should compute the set of those variables that must be $true$ in order to make the formula $true$
  - E.g, if we know that x and y must be $true$ and $x \wedge y \to z$ is a clause of the formula, then z must be $true$
  - Thus the system should follow here „edges" of the form $x \wedge y \to z$

P upper bound for a restricted variant of P systems with AM's

- Giving polynomial time upper bound on the power of P systems with AM's is hard if *both division and dissolution rules* are allowed (even if the rules have no polarizations).
- Example: using the rules $[a] \rightarrow [a_1][a_2], [b] \rightarrow [b_1][b_2], [c] \rightarrow [c_1][c_2]$ on the membrane $[a, b, c]$ yields $2^3 = 8$ different membranes
  - Storing the representation of each membrane needs exponential space
- P upper bound is given when polarization, evolution and communication is not allowed, and the initial membrane structure is a sequence single path [2009, Woods et al.]
  - Object division graph is used to follow the possible divisions for a given object and membrane label
  - The numbers of different objects in a membrane are stored in a vector

P upper bound for a restricted variant of P systems with AM's

- We consider P systems with AM's without polarization, evolution and communication
- We propose a method for representing exponentially many different membranes using polynomial space
- Recall: the representation is hard only for the elementary membranes
- Consider the rules
  - $[a] \to [a_1][a_2], [b] \to [b_1][b_2], [c] \to [c_1][c_2]$ and
  - $[a_2] \to \bar{a}$ and $[c_1] \to \bar{c}$
  - Let $C = [a, b, c, d]$
  - The representation of C after the
    - 1st step: $(\, a_1 \mid a_2 \,)\, [b, c, d]$
    - 2nd step: $[a_1, b, c, d]$ (the multiset $\{\bar{a}, b, c, d\}$ should be added to the representation of objects in the parent)

P upper bound for a restricted variant of P systems with AM's

- The representation of C after the
  - 3rd step: $(\, b_1 \mid b_2\,)\ [a_1, c, d]$
  - 4th step: $(\, b_1 \mid b_2\,)(c_1 \mid c_2)\ [a_1, d]$
  - 5th step: $(\, b_1 \mid b_2\,)\ [c_2, a_1, d]$ (the multiset represented by $(\, b_1 \mid b_2\,)\ [\bar{c}, a_1, d]$ should be added to the representation of objects in the parent)
- At every step
  - the new representation of the elementary membranes and
  - the representation of the objects in the parents can be computed in polynomial time
  - (no formal proof yet)
- If the construction works, the next step is to extend it to out-communication rules

Summary

- If we can prove the correctness of the constructions, then
  - the power of P systems with no evolution, dissolution, division and in-communication rules characterize the complexity class P,
  - the power of P systems with no evolution and communication rules lower bounded by P, and
  - the power of P systems with no polarization, evolution and communication rules is upper bounded by P

Thank you!