Space Filling Curves - Applications: Contour approximation (an idea, more questions)

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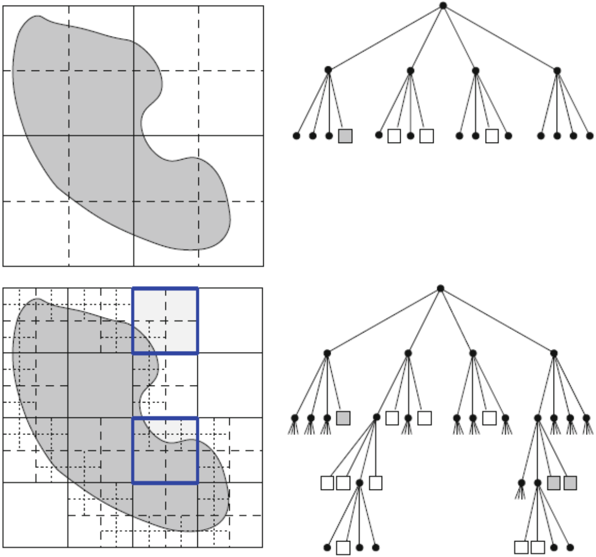
Proposed goal from prev work

1. Generation of SFCs (linear and array) using P systems
2. Open: to use P systems to model more complex applications of SFCs
3. Older: tissue P systems for 2D picture repres/generation

Representing 2D objects - quadtrees



Hilbert Traversal of Quadtrees - Locality

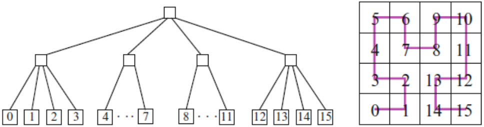
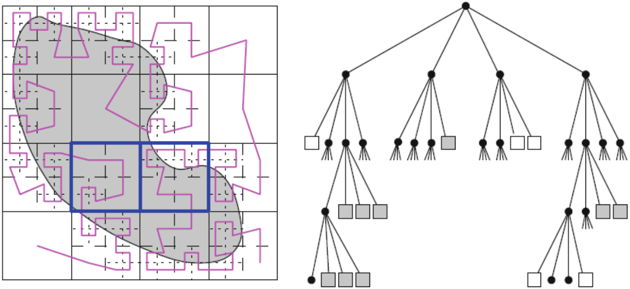
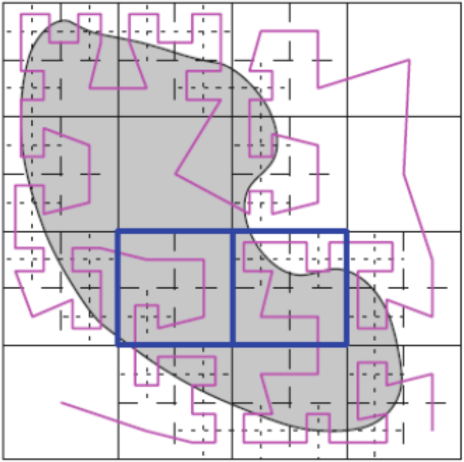


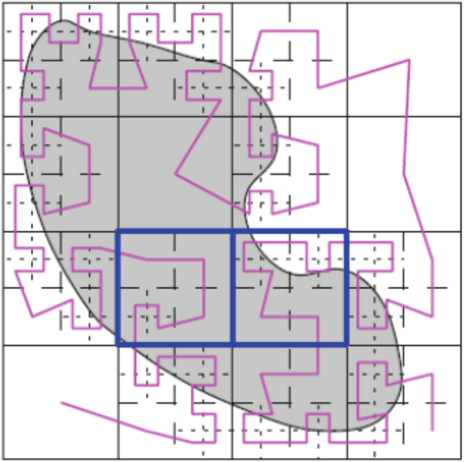
Fig. 1.5 Quadtree representation of a regular 4 × 4 grid, and a sequential order that avoids jumps



Contour approximation



Contour approximation



Luis' Question: HOW?

Remember the Hilbert curve

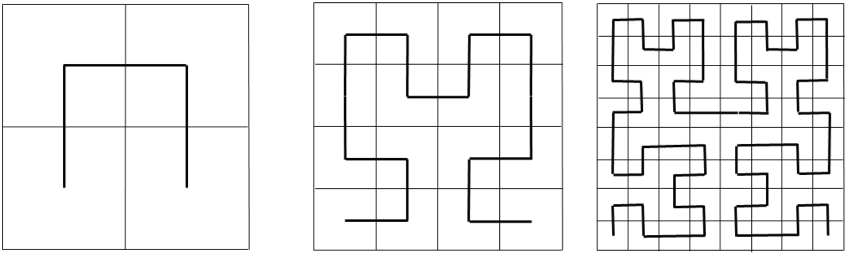


Figure: The first three patterns of the sequence defining the Hilbert curve

non-terminals 1×1 arrays $\bar{N} = \{Ud, Ur, Ru, Rl, Ld, Lr, Du, Dl\}$
Γ the array morphism of the eight rewriting rules bellow:

$$U* \to \begin{array}{cc} Ur & Ud \\ Ru & L* \end{array} \qquad with\ * = d, r \tag{1}$$

$$R* \to \begin{array}{cc} D* & Rl \\ Ur & Ru \end{array} \qquad with\ * = u, l \tag{2}$$

$$L* \to \begin{array}{cc} Ld & Dl \\ Lr & U* \end{array} \qquad with\ * = d, r \tag{3}$$

$$D* \to \begin{array}{cc} R* & Ld \\ Du & Dl \end{array} \qquad with\ * = u, l \tag{4}$$

F the array morphism of the eight rewriting rules below:

$$Ud \to d, Ur \to r, Ld \to d, Lr \to r, Ru \to u, Rl \to l, Du \to u, Dl \to l \tag{5}$$

$F(\Gamma^n(Ur)) =$ the nth Hilbert word $H_n r$ in array representation.

Add 'empty' array rule

$$U^1 * \rightarrow \begin{matrix} Ur & Ud \\ Ru & L* \end{matrix} \qquad with * = d, r \tag{6}$$

$$R^1 * \rightarrow \begin{matrix} D* & Rl \\ Ur & Ru \end{matrix} \qquad with * = u, l \tag{7}$$

$$L^1 * \rightarrow \begin{matrix} Ld & Dl \\ Lr & U* \end{matrix} \qquad with * = d, r \tag{8}$$

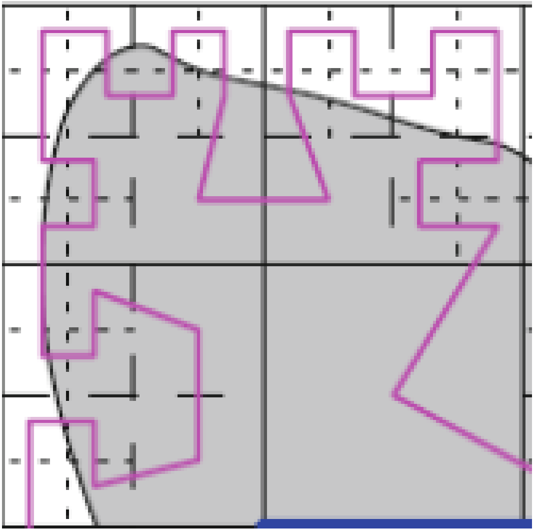
$$D^1 * \rightarrow \begin{matrix} R* & Ld \\ Du & Dl \end{matrix} \qquad with * = u, l \tag{9}$$

$$U^0 * = \#^0 \rightarrow \begin{matrix} \#^0 & \#^0 \\ \#^0 & \#^0 \end{matrix} = \#^1 \qquad with * = d, r \tag{10}$$

$$R^0 * \rightarrow \#^0 \qquad with * = u, l \tag{11}$$

$$L^0 * \rightarrow \#^0 \qquad with * = d, r \tag{12}$$

$$D^0 * \rightarrow \#^0 \qquad with * = u, l \tag{13}$$



Membrane division rules – to generate a tissue P sys

$$[U^1*] \rightarrow \begin{matrix} [Ur] & [Ud] \\ [Ru] & [L*] \end{matrix} \qquad with * = d, r \tag{14}$$

$$[R^1*] \rightarrow \begin{matrix} [D*] & [Rl] \\ [Ur] & [Ru] \end{matrix} \qquad with * = u, l \tag{15}$$

$$[L^1*] \rightarrow \begin{matrix} [Ld] & [Dl] \\ [Lr] & [U*] \end{matrix} \qquad with * = d, r \tag{16}$$

$$[D^1*] \rightarrow \begin{matrix} [R*] & [Ld] \\ [Du] & [Dl] \end{matrix} \qquad with * = u, l \tag{17}$$

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$$\#^0 \to \begin{matrix} \#^0 & \#^0 \\ \#^0 & \#^0 \end{matrix} = \#^1 \tag{18}$$

$$[U^1*] \rightarrow \begin{matrix} [Ur] & [Ud] \\ [Ru] & [L*] \end{matrix}$$

$$[U^1*] \rightarrow \begin{matrix} [Ur] & [Ud] \\ [Ru] & [L*] \end{matrix} \rightsquigarrow \begin{matrix} [U^1r] & [U^1d] \\ [R^1u] & [L^1*] \end{matrix} \rightarrow$$

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$$\rightarrow \begin{matrix} [Ur] & [Ud] & [Ur] & [Ud] \\ [Ru] & [Lr] & [Ru] & [Ld] \\ [Du] & [Rl] & [Ld] & [Dl] \\ [Ur] & [Ru] & [Lr] & [U*] \end{matrix}$$

$$[U^1*] \rightarrow \begin{matrix} [Ur] & [Ud] \\ [Ru] & [L*] \end{matrix} \rightsquigarrow \begin{matrix} [U^1r] & [U^1d] \\ [R^1u] & [L^1*] \end{matrix} \rightarrow$$

$$\rightarrow \begin{matrix} [Ur] & [Ud] & [Ur] & [Ud] \\ [Ru] & [Lr] & [Ru] & [Ld] \\ [Du] & [Rl] & [Ld] & [Dl] \\ [Ur] & [Ru] & [Lr] & [U*] \end{matrix} \rightsquigarrow \begin{matrix} [U^1r] & [U^1d] & [U^1r] & [U^0d] \\ [R^1u] & [L^0r] & [R^1u] & [L^0d] \\ [D^1u] & [R^1l] & [L^1d] & [D^1l] \\ [U^0r] & [R^1u] & [L^1r] & [U^1*] \end{matrix} \rightarrow$$

$$
\rightarrow \begin{matrix}
[Ur] & [Ud] & [Ur] & [Ud] & [Ur] & [Ud] & [\#^0] & [\#^0] \\
[Ru] & [Lr] & [Ru] & [Ld] & [Ru] & [Lr] & [\#^0] & [\#^0] \\
[Du] & [Rl] & [\#^0] & [\#^0] & [Du] & [Rl] & [\#^0] & [\#^0] \\
[Ur] & [Ru] & [\#^0] & [\#^0] & [Ur] & [Ru] & [\#^0] & [\#^0] \\
[Ru] & [Ld] & [Dl] & [Rl] & [Ld] & [Dl] & [Rl] & [Ld] \\
[Du] & [Dl] & [Ur] & [Ru] & [Lr] & [Ud] & [Du] & [Dl] \\
[\#^0] & [\#^0] & [Du] & [Rl] & [Ld] & [Dl] & [Ur] & [Ud] \\
[\#^0] & [\#^0] & [Ur] & [Ru] & [Lr] & [Ur] & [Ru] & [L*]
\end{matrix} \rightsquigarrow
$$

$$
\to
\begin{matrix}
[Ur] & [Ud] & [Ur] & [Ud] & [Ur] & [Ud] & [\#^0] & [\#^0] \\
[Ru] & [Lr] & [Ru] & [Ld] & [Ru] & [Lr] & [\#^0] & [\#^0] \\
[Du] & [Rl] & [\#^0] & [\#^0] & [Du] & [Rl] & [\#^0] & [\#^0] \\
[Ur] & [Ru] & [\#^0] & [\#^0] & [Ur] & [Ru] & [\#^0] & [\#^0] \\
[Ru] & [Ld] & [Dl] & [Rl] & [Ld] & [Dl] & [Rl] & [Ld] \\
[Du] & [Dl] & [Ur] & [Ru] & [Lr] & [Ud] & [Du] & [Dl] \\
[\#^0] & [\#^0] & [Du] & [Rl] & [Ld] & [Dl] & [Ur] & [Ud] \\
[\#^0] & [\#^0] & [Ur] & [Ru] & [Lr] & [Ur] & [Ru] & [L*]
\end{matrix}
\leadsto
$$

$$
\leadsto
\begin{matrix}
[Ur] & [Ud] & [Ur] & [Ud] & [U^0 r] & [U^0 d] & [\#^0] & [\#^0] \\
[Ru] & [L^0 r] & [R^0 u] & [Ld] & [Ru] & [L^0 r] & [\#^0] & [\#^0] \\
[Du] & [R^0 l] & [\#^0] & [\#^0] & [Du] & [R^0 l] & [\#^0] & [\#^0] \\
[Ur] & [R^0 u] & [\#^0] & [\#^0] & [Ur] & [R^0 u] & [\#^0] & [\#^0] \\
[Ru] & [Ld] & [D^0 l] & [R^0 l] & [Ld] & [Dl] & [Rl] & [L^0 d] \\
[D^0 u] & [Dl] & [Ur] & [R^0 u] & [L^0 r] & [U^0 d] & [Du] & [D^0 l] \\
[\#^0] & [\#^0] & [Du] & [Rl] & [L^0 d] & [D^0 l] & [Ur] & [U^0 d] \\
[\#^0] & [\#^0] & [U^0 r] & [Ru] & [Lr] & [Ur] & [Ru] & [L^0 *]
\end{matrix}
\to \cdots
$$

Labels

$$[U^1*] \rightarrow \begin{matrix} [Ur]_{nw} & [Ud]_{ne} \\ [Ru]_{sw} & [L*]_{se} \end{matrix} \qquad with * = d, r \tag{19}$$

$$[R^1*] \rightarrow \begin{matrix} [D*]_{nw} & [Rl]_{ne} \\ [Ur]_{sw} & [Ru]_{se} \end{matrix} \qquad with * = u, l \tag{20}$$

$$[L^1*] \rightarrow \begin{matrix} [Ld]_{nw} & [Dl]_{ne} \\ [Lr]_{sw} & [U*]_{se} \end{matrix} \qquad with * = d, r \tag{21}$$

$$[D^1*] \rightarrow \begin{matrix} [R*]_{nw} & [Ld]_{ne} \\ [Du]_{sw} & [Dl]_{se} \end{matrix} \qquad with * = u, l \tag{22}$$

Labels

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Alternative - labels in binary (many interesting codes!)

1. Started from ...
    1. to use P systems to model more complex applications of SFCs
    2. tissue P systems for 2D picture repres/generation

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   1. to use P systems to model more complex applications of SFCs
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   1. using SFCs to generate tissue P systems

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    1. using SFCs to generate tissue P systems
    2. 2-level P systems (combining the 2)

1. Started from ...
    1. to use P systems to model more complex applications of SFCs
    2. tissue P systems for 2D picture repres/generation
2. arrived at
    1. using SFCs to generate tissue P systems
    2. 2-level P systems (combining the 2)
    3. more questions
        1. labels, history...
        2. how to input/decide (0 or 1)?

Thank you!