Multidimensional Descriptional Complexity of P Systems *

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Abstract

Membrane Computing is a branch of Natural Computing which starts from the assumption that the processes taking place in the compartmental structure of a living cell can be interpreted as computations. The description of the complexity of the computations of the membrane devices (P systems) is a hard task which goes beyond the usual parameters of time and space. This is especially hard in the case of P systems where the number of membranes increases along the computation, via division or creation of membranes. In this paper we show that a four-dimensional carpet can be a useful tool to describe and compare evolutions of P systems, even in such cases.

1 Introduction

Membrane Computing, introduced by Gh. Păun in [11], is a cross-disciplinary field where computer scientists, biologists, formal linguists and complexity theoreticians are getting involved, enriching each others with results, open problems and promising new research lines. Indeed, Membrane Computing has been selected in February 2003 by the Institute for Scientific Information, USA, as a fast *Emerging Research Front* in Computer Science, and [10] was mentioned in the ISI web page (`http://esi-topics.com/erf/october2003.html`) as a highly cited paper in October 2003.

This new non-deterministic model of computation abstracts the processes taking place in the compartmental structure of a living cell, interpreting them as computing operations. The devices of this model are called *membrane systems* or *P systems*. Roughly speaking, a P system consists of a cell-like membrane structure, in the compartments of which one places multisets of objects which evolve according to given rules in a synchronous non-deterministic maximally parallel manner[1].

The evolution of a P system is a complex process where (possibly) a large number of symbol-objects, membranes and rules are involved. Furthermore, if we work with models

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[1]A layman-oriented introduction can be found in [13], a comprehensive monograph in [12], and the latest information about P systems is available at [16].

where the number of membranes can increase along the computation, via creation or division of membranes, then it becomes more complicated to describe the complexity of the computational process. Such models have actually been investigated largely in the literature as their ability to generate an exponential number of membranes in polynomial time (making use of their intrinsic parallelism) makes them powerful tools to address **NP**-complete problems.

The complexity in *time* (number of cellular steps) of the solutions obtained in this way is polynomial, but it is clear that the time is not the unique variable that we need to consider in order to evaluate the complexity of such processes. Ciobanu, Păun and Ştefănescu presented in [1] a new way to describe the complexity of a computation in a P system, the so-called *Sevilla Carpet*, which is an extension of the notion of Szilard language from grammars to the case when several rules are used at the same time.

In [3], new parameters for the study of the descriptive complexity of P systems were introduced and several examples of the graphical representation and the utility of these parameters for comparing different solutions for a same problem were provided.

In this paper we extend the definition of Sevilla carpets to describe the computations of P systems in a new way. We present a four dimensional manifold which can be used for a better understanding of the complexity of a P system. The graphical representation of this four-dimensional manifold is carried out via projections on three dimensional spaces.

The paper is organized as follows. First we give a short overview on membrane systems (focusing on the models with active membranes and with membrane creation), and on how they can be used as devices that solve problems. In Section 3 we recall the Sevilla carpets with the parameters associated with them, and we generalize them to the four dimensional case in Section 4. In order to illustrate the usefullness of this generalization, in Section 5 we present a comparative study of the descriptional complexity of two cellular solutions to the Subset Sum problem, one of them using active membranes and the other one using membrane creation. Finally, some concluding remarks are given in Section 6.

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