Formal Frameworks

An Introduction

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Personal stats

My first brainstorming: BWMC 2013, 10 years ago!

So far: 8 in person BWMC + online BWMC2022.

Thank you!

Looking forward to the future!

Riddle

What is a formal framework?

Riddle

What is a formal framework?

1. The topic of this talk.
2. Something a P scientist does at least once in his or her career.

😉

Formal framework in P systems

A complete rulebook of an abstract object.

A set of formal definitions describing the structure[1,2] and the behavior[3,4] of a class of objects $\mathcal{X}$:

1. Which objects belong to $\mathcal{X}$?
2. Which objects do not belong to $\mathcal{X}$?
3. What can objects in $\mathcal{X}$ do?
4. What can objects in $\mathcal{X}$ not do?

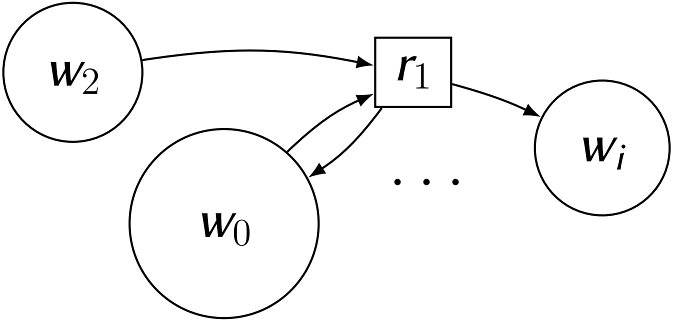
My favorite example

📕 *Rudolf Freund, Sergey Verlan: A Formal Framework for Static (Tissue) P Systems. Workshop on Membrane Computing 2007: 271-284*

Many other formal frameworks exist.

Networks of cells

$$\Pi = (n, V, w, Inf, R)$$



- membranes are not nested
- rules are global
- topology is induced by rules

Zoom in on the rules

$$\left(X \longrightarrow Y;\ P, Q\right)$$

- $X \longrightarrow Y = (x_1, 1), \ldots, (x_n, n) \to (y_1, 1), \ldots, (y_n, n)$
- $P = (p_1, 1), \ldots, (p_n, n)$
- $Q = (q_1, 1), \ldots, (q_n, n)$
- p_1 and q_1: finite sets of multisets

Applicability, $\forall i$:

$$\forall p \in p_i : p \subseteq w_i \ \wedge\ \forall q \in p_i : q \not\subseteq w_i \ \wedge\ x_i \subseteq w_i$$

First conclusions

This formal framework captures/generalizes:

- multiset rewriting
- communication
- connection topology
- the environment

More on applicability

$$Appl(\Pi, C)$$

The set of all multiset of rules of Π applicable in configuration C, defined rigorously and constructively (by an algorithm).

$$Apply(\Pi, C, R'), \qquad R' \in Appl(\Pi, C)$$

Remove all left-hand sides of R', add back all right-hand sides in R'.

Derivation modes

$$Appl(\Pi, C, asyn) = Appl(\Pi, C)$$

any applicable multiset of rules can be applied

$$Appl(\Pi, C, sequ) = \{R' \in Appl(\Pi, C) \mid |R'| = 1\}$$

only multisets of size 1 can be applied

$$Appl(\Pi, C, max) = \{R' \in Appl(\Pi, C) \mid \nexists R'' \in Appl(\Pi, C) : R' \subseteq R''\}$$

only non-extendable multisets of rules can be applied

$\dots$

Why formal frameworks?

Clean common language

I tell you what my object is, and you understand 100%.

- rule shapes
- rule applicability
- derivation modes
- . . .

Detect bugs

Minimal parallelism : “if at least a rule from a set of rules associated with a membrane or a region can be used, then at least one rule from that membrane or region must be used, without any other restriction.”

Multiple different interpretations have been used.

The formal framework allows capturing, analyzing, and comparing them.

Gabriel Ciobanu, Linqiang Pan, Gheorghe Păun, Mario J. Pérez-Jiménez: P systems with minimal parallelism. Theor. Comput. Sci. 378(1): 117-130 (2007)

Formalize intuitive similarities

Interacting with the membranes is kind of like interacting with objects.

$$
\begin{aligned}
[a \to bc]_h \quad &\Rightarrow \quad (k, a) \to (k, bc); (k, h) \\
a[\,]_h \to [b]_{h'} \quad &\Rightarrow \quad (k', a)(k, h) \to (k, bh')
\end{aligned}
$$

k and k' are the unique names of the membrane with label h to which the rule is applied.

Sergey Verlan: Using the Formal Framework for P Systems. Int. Conf. on Membrane Computing 2013: 56-79

Invent new ingredients

$$\begin{aligned} Appl(\Pi, C, max) = \{&R' \in Appl(\Pi, C) \\ &\mid \nexists R'' \in Appl(\Pi, C) : R' \subseteq R''\} \end{aligned}$$

Note: $|R'|$ is not necessarily maximized!

$$\Downarrow$$

$$\begin{aligned} Appl(\Pi, C, max_{rules}) = \{&R' \in Appl(\Pi, C) \\ &\mid \nexists R'' \in Appl(\Pi, C) : R' < R''\} \end{aligned}$$

📕 Artiom Alhazov, Rudolf Freund, Sergiu Ivanov: When catalytic P systems with one catalyst can be computationally complete. J. Membr. Comput. 3(3): 170-181 (2021)

Enable simulators

Simulator = computer program acting on data structures

Data structures = formalization of abstract objects

Formal frameworks!

This is what any P scientist does at least once!

How to use formal frameworks?

Rules of thumb

Use a formal framework as a starting point for designing a P system variant.

Use a formal framework to interpret and understand the features of a P system variant.

A formal framework is not a substitute for a P system variant! It is a tool for better understanding it.

Uses of formal frameworks

- Clean common language
- Detect bugs
- Formalize intuitive similarities
- Invent new ingredients
- Enable simulators